

Mixing and lifetimes of b -hadrons

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Abstract. We review the status of mixing and lifetimes of b -hadrons. We will show that $\Delta\Gamma/\Delta M$, a_{sl} and ϕ are better suited to search for new physics effects than ΔM alone, because of our poor knowledge of the decay constants. The theoretical precision in the determination of Γ_{12}/M_{12} - which contains all information on $\Delta\Gamma/\Delta M$, a_{sl} and ϕ - can be tested directly by investigating the lifetimes of b -hadrons, because both quantities rely on the same theoretical footing. In particular we will also present a numerical estimate for the lifetime of the Ξ_b -baryon.

INTRODUCTION - THEORETICAL TOOLS

In this section we briefly discuss the principles of the calculation of physical quantities like lifetimes of b -hadrons τ , the mass difference in the neutral B-meson system ΔM , the decay rate difference in the neutral B-meson system $\Delta\Gamma$, the semi-leptonic CP-asymmetry a_{sl} and the mixing phase ϕ . These quantities are currently measured at the B-factories and at the TeVatron (see e.g. [1]) and they will be measured at the LHC [2] or at a Super-B-factory [3] with high precision. They are defined as (see e.g. [4] for more details):

$$\frac{1}{\tau} = \sum_X \Gamma(B \rightarrow X), \quad (1)$$

$$\Delta M = M_H - M_L = 2|M_{12}|, \quad \Delta\Gamma = \Gamma_L - \Gamma_H = 2|\Gamma_{12}|\cos(\phi), \quad (2)$$

$$a_{sl} = \Im\left(\frac{\Gamma_{12}}{M_{12}}\right), \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right). \quad (3)$$

For the lifetimes one has to sum the decay rates into all possible final states X . The mixing stems from so-called box diagrams (see below). M_{12} is the dispersive part (sensitive to heavy internal particles) and Γ_{12} is the absorptive part (sensitive to light internal particles) of these box diagrams. In the standard model $\tau = 1/\Gamma$, M_{12} and Γ_{12} are given by the following diagrams (as an example we draw the diagrams for the B_s -meson):

$$\Gamma = \int \sum_X \left| \begin{array}{c} b \quad c, u \\ \text{---} \bullet \text{---} \\ | \\ \text{W} \\ | \\ \bullet \text{---} \bar{c}, \bar{u} \\ | \\ s, d \end{array} \right|^2, \quad M_{12} = \begin{array}{c} b \quad t \quad s \\ \text{---} \bullet \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{W} \quad \text{W} \quad \text{W} \\ | \quad | \quad | \\ \bar{s} \quad \bar{t} \quad \bar{b} \end{array}, \quad \Gamma_{12} = \begin{array}{c} b \quad c, u \quad s \\ \text{---} \bullet \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{W} \quad \text{W} \quad \text{W} \\ | \quad | \quad | \\ \bar{s} \quad \bar{c}, \bar{u} \quad \bar{b} \end{array}$$

All these quantities are triggered by weak decays, in particular by the exchange of heavy W -bosons and the top-quark. Using the fact that these particles are much heavier than the b -quark ($m_t, m_W \gg m_b$) one can integrate them out by performing an operator product expansion (OPE I), see e.g. [5] for a nice introduction. In the resulting effective theory the standard model diagrams are rewritten in a product of perturbative Wilson coefficients and new operators, they now look like that:

$$\Gamma = \int \sum_X \left| \begin{array}{c} \text{Diagram 1: } b \text{ quark line entering a vertex (marked with a cross), from which } c, u \text{ and } \bar{c}, \bar{u} \text{ quarks emerge. Another vertex (marked with a cross) receives } s, d \text{ quarks and emits } \bar{s}, \bar{d} \text{ quarks.} \end{array} \right|^2, \quad M_{12} = \begin{array}{c} \text{Diagram 2: } b \text{ and } \bar{s} \text{ quarks enter a vertex (marked with a cross), and } s \text{ and } \bar{b} \text{ quarks emerge.} \end{array}, \quad \Gamma_{12} = \begin{array}{c} \text{Diagram 3: } b \text{ and } \bar{s} \text{ quarks enter a vertex (marked with a cross), and } s \text{ and } \bar{b} \text{ quarks emerge. A loop of } c, u \text{ and } \bar{c}, \bar{u} \text{ quarks connects this vertex to another vertex (marked with a cross) which receives } s, d \text{ quarks and emits } \bar{s}, \bar{d} \text{ quarks.} \end{array}$$

The vertices in the diagrams for Γ and Γ_{12} are effective four-quark operators with $\Delta B = 1$, while the vertex in the diagram for M_{12} is an effective four-quark operator with $\Delta B = 2$. For M_{12} we have now already the final local operator, whose matrix element has to be determined with some non-perturbative QCD-method.

As a next step we rewrite the expression for Γ in a form that is almost identical to the one of Γ_{12} . With the help of the optical theorem Γ can be rewritten (diagrammatically: a mirror reflection on the right end of the decay diagram followed by all possible Wick contractions of the quark lines) in

$$\Gamma = \begin{array}{c} \text{Diagram 4: } b \text{ quark line enters a vertex (marked with a cross), from which } c, u \text{ and } \bar{c}, \bar{u} \text{ quarks emerge. Another vertex (marked with a cross) receives } s, d \text{ quarks and emits } \bar{s}, \bar{d} \text{ quarks. A horizontal line with } \bar{s} \text{ quark continues from the bottom vertex.} \end{array} + \dots + \begin{array}{c} \text{Diagram 5: } b \text{ and } \bar{s} \text{ quarks enter a vertex (marked with a cross), and } s \text{ and } \bar{b} \text{ quarks emerge. A loop of } c, u \text{ and } \bar{c}, \bar{u} \text{ quarks connects this vertex to another vertex (marked with a cross) which receives } s, d \text{ quarks and emits } \bar{s}, \bar{d} \text{ quarks.} \end{array} + \dots$$

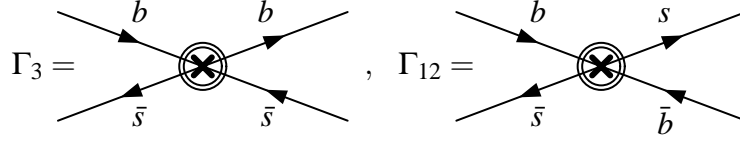
$\Gamma_0 \qquad \qquad \qquad \Gamma_3$

The first term ($=: \Gamma_0$) corresponds to the decay of a free b -quark, see e.g. [6, 7, 8] and references therein for some applications. This term gives the same contribution to all b -hadrons. The lifetime differences we are interested in will only appear in subleading terms of this expansion like the second diagram ($=: \Gamma_3$), which looks very similar to the diagram for Γ_{12} . Counting the mass dimensions of the external lines one can write formally an expansion of the total decay rate in inverse powers of the heavy quark mass m_b :

$$\Gamma = \Gamma_0 + \frac{\Lambda}{m_b} \Gamma_1 + \frac{\Lambda^2}{m_b^2} \Gamma_2 + \frac{\Lambda^3}{m_b^3} \Gamma_3 + \dots \quad (4)$$

However the expressions for Γ_i and Γ_{12} are still non-local, so we perform a second OPE (OPE II) using the fact that the b -quark mass is heavier than the QCD scale

($m_b \gg \Lambda_{QCD}$). The OPE II is called the heavy quark expansion (HQE) [9, 10, 11, 12, 13, 14, 15, 16]. The resulting diagrams for Γ_3 and Γ_{12} look like the final diagram for M_{12} :



Now we are left with local four-quark operators ($\Delta B = 0$ for τ and $\Delta B = 2$ for Γ_{12}). The non-perturbative matrix elements of these operators are expressed in terms of decay constants f_B and bag parameters B . In the standard model one gets one operator for M_{12} , two independent operators for Γ_{12} - including the operator that appears in M_{12} - and e.g. four operators for $\tau(B^+)/\tau(B_d)$ ¹ - in extensions of the standard model typically more operators arise.

At this stage we would like to make some comments:

- One can show that in the end no corrections of order $1/m_b$ survive in the total decay rate in Eq. (4).
- Γ and Γ_{12} are expected to be almost free from possible new physics contributions, since only light internal particles contribute, while M_{12} might easily have large contributions from new physics effects. Since allowed new physics contributions to Γ and Γ_{12} are smaller than the QCD uncertainties we neglect them in the following.
- The OPE II seems to be theoretically less justified than the OPE I ($m_W/m_b \approx 17...19 > 4...10 \approx m_b/\Lambda_{QCD}$), but the HQE can be tested directly by comparing experiment and theory for the lifetimes.
- In all the diagrams shown in this section perturbative QCD-corrections have to be included! These corrections to the Wilson coefficients turned out to be quite sizeable.

Summarizing one can state: the HQE represents a systematic expansion, which can in principle be tested by the lifetimes - in that sense it is not a model like the quark model.

INTRODUCTION - MOTIVATION

Besides testing our understanding of QCD and determining the standard model parameters the search for new physics effects is a basic motivation for the study of the mixing quantities. Since M_{12} is sensitive to heavy new internal particles one might start with ΔM_s , which is proportional to $f_{B_s}^2 B$ (see next section). Unfortunately f_{B_s} is hardly known. To visualise our current unsatisfactory knowledge of the precise value of the decay constant, we have taken some recent numerical values for f_{B_s} from the literature [17, 18, 19, 20, 21] and calculated the corresponding value of the mass difference ΔM_s .

¹ This statements hold only at order $1/m_b^3$.

f_{B_s}	N_F	ΔM_s	deviation from experiment
$193 \pm 06 \text{ MeV [17]}$	0	$12.5 \pm 1.4 \text{ ps}^{-1}$	-3.9σ
$205 \pm 32 \text{ MeV [18]}$	2	$14.1 \pm 4.6 \text{ ps}^{-1}$	-0.8σ
$259 \pm 26 \text{ MeV [19]}$	3	$22.5 \pm 5.0 \text{ ps}^{-1}$	$+0.9 \sigma$
$297 \pm 14 \text{ MeV [20]}$	2	$30.0 \pm 3.9 \text{ ps}^{-1}$	$+3.1 \sigma$
$341 \pm 32 \text{ MeV [21]}$	2	$39.0 \pm 8.2 \text{ ps}^{-1}$	$+2.6 \sigma$

(5)

Depending on your favorite lattice collaboration you can arrive at theory predictions that are smaller, are equal or are higher than the experimental value for the mass difference ΔM_s . This unfortunate situation might be called the **decay constant problem**. Here clearly more work has to be done to settle this ignorance and moreover the error estimates have to be done with much care. In the analysis in [4] we use the conservative estimate $f_{B_s} = 240 \pm 40 \text{ MeV}$.

In order to circumvent the decay constant problem one might try to determine the ratio $\Delta M_s / \Delta M_d$. Here the ratio $|V_{ts}^2 / V_{td}^2| \cdot f_{B_s}^2 B_{B_s} / (f_{B_d}^2 B_{B_d})$ arises. Although the ratio of the non-perturbative parameters is claimed to be theoretically better under control, one is still left with the uncertainty in the CKM elements, which is of the order of 40%.

In the ratio Γ_{12} / M_{12} the decay constant and the bag parameter B from M_{12} cancel completely, schematically one gets

$$\frac{\Gamma_{12}}{M_{12}} = a + b \frac{B_X}{B} + \mathcal{O}\left(\frac{1}{m_b^4}\right). \quad (6)$$

We get a term that is completely free of any non-perturbative uncertainties (a) and a term that depends on the ratio of two bag parameters (b). If $a > b$ and if the remaining uncertainties are under control then Γ_{12} / M_{12} might be an ideal quantity to search for new physics. Moreover the accuracy in the determination of this ratio can be tested directly via the lifetimes, which root on the same theoretical footing.

STATE OF THE ART

In this section we summarize the current status in the theoretical determination of the lifetimes of the b -hadrons and the mixing quantities.

The mass difference - M_{12}

Calculating the box diagram with internal top quarks one obtains

$$M_{12,q} = \frac{G_F^2}{12\pi^2} (V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B. \quad (7)$$

The Inami-Lim function $S_0(x_t = \bar{m}_t^2 / M_W^2)$ [22] is the result of the box diagram without any gluon corrections. The NLO QCD correction is parameterized by $\hat{\eta}_B \approx 0.84$ [23].

The non-perturbative matrix element is parameterized by the bag parameter B and the decay constant f_B .

The decay rate difference - Γ_{12}

The calculation of Γ_{12} is a little bit more involved since a second OPE has to be performed. Γ_{12} can be expanded as

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{(0)} + \dots \right) + \dots \quad (8)$$

The $1/m_b$ -corrections ($\Gamma_4^{(0)}$) were determined in [24] and they turned out to be quite sizeable. NLO QCD-corrections were done for the first time in [25], they also were quite large. At that time no lattice results were available for all appearing four-quark operators, so no real numerical prediction could be made. The first numerical estimate including NLO-QCD corrections and non-perturbative determinations of the appearing four-quark operators was given in [26]. Five years later the QCD-corrections were confirmed and also subleading CKM structures were included [27, 28]. Unfortunately it turned out that $\Delta\Gamma$ is not well-behaved [29]. All corrections are unexpectedly large and they go in the same direction. This problem could be solved by introducing a new operator basis [4]. As an illustration of the improvement we show the expressions for Γ_{12}/M_{12} in the old and the new basis:

$$\frac{\Delta\Gamma_s}{\Delta M_s}^{\text{Old}} = 10^{-4} \cdot \left[0.9 + 40.9 \frac{B'_S}{B} - 25.0 \frac{B_R}{B} \right], \quad (9)$$

$$\frac{\Delta\Gamma_s}{\Delta M_s}^{\text{New}} = 10^{-4} \cdot \left[46.2 + 10.6 \frac{B''_S}{B} - 11.9 \frac{B_R}{B} \right]. \quad (10)$$

Now the term that is completely free of any non-perturbative uncertainties is numerical dominant. Moreover the $1/m_b$ -corrections became smaller and undesired cancellations are less pronounced. For more details we refer the reader to [4]. Currently also $1/m_b$ -corrections for the subleading CKM structures in Γ_{12} [30] and $1/m_b^2$ -corrections for $\Delta\Gamma_s$ [31] are available - they are relatively small.

Lifetimes

The lifetime ratio of two b -hadrons can be written as

$$\frac{\tau_1}{\tau_2} = 1 + \frac{\Lambda^2}{m_b^2} \Gamma_2 + \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{(0)} + \dots \right) + \dots \quad (11)$$

Γ_2 vanishes e.g. in τ_{B^+}/τ_{B_d} , τ_{B_s}/τ_{B_d} and $\tau_{\Xi_b^+}/\tau_{\Xi_b^0}$ but it survives in $\tau_{\Lambda_b}/\tau_{B_d}$. The sizeable NLO QCD-corrections to the lifetime ratios ($\Gamma_3^{(1)}$) were determined in [32, 33];

$1/m_b$ -corrections ($\Gamma_4^{(0)}$) and $1/m_b^2$ -corrections ($\Gamma_5^{(0)}$) were calculated in [34] - they are negligible for τ_{B^+}/τ_{B_d} and τ_{B_s}/τ_{B_d} , but they might be sizeable for $\tau_{\Lambda_b}/\tau_{B_d}$.

NUMERICAL RESULTS

Lifetimes

The theoretically best investigated lifetime ratio is τ_{B^+}/τ_{B_d} . One obtains [32, 33]

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.063 \pm 0.027. \quad (12)$$

NLO-QCD corrections turned out to be important, while subleading $1/m_b$ -corrections are negligible. Some care has to be taken with the arising matrix elements of the four-quark operators: it turned out that the Wilson coefficients of the color-suppressed operators are numerically enhanced, see [32]. But the matrix elements of these operators are only known with large relative errors. Currently two determinations on the lattice are available [35, 36].

For τ_{B_s}/τ_{B_d} large cancellations occur so the ratio is expected to be very close to one [24, 33]

$$\frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01. \quad (13)$$

Predictions for the Λ_b have to be taken with more care. In that case the NLO-QCD corrections are not complete and only preliminary lattice values [37] are available. A typical value quoted in the literature [38] is

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.88 \pm 0.05. \quad (14)$$

The lifetime of the doubly heavy meson B_c has been investigated e.g. in [39], but only in LO QCD.

$$\tau(B_c)_{\text{LO}} = 0.52^{+0.18}_{-0.12} \text{ ps}.$$

In addition to the b-quark now also the c-quark can decay, giving rise to the biggest contribution to the total decay rate.

An interesting quantity is the lifetime ratio of the Ξ_b -baryons, which was investigated in NLO-QCD in [32]. This quantity can in principle be determined as precise as τ_{B^+}/τ_{B_d} ($\pm 3\%$). However, up to now the matrix elements for the Ξ_b baryons are not available. Assuming that the matrix elements for Ξ_b are equal to the ones of Λ_b we can give a rough estimate for the expected lifetime ratio. In order to get rid of unwanted $s \rightarrow u$ -transitions we define (following [32])

$$\frac{1}{\bar{\tau}(\Xi_b)} = \bar{\Gamma}(\Xi_b) = \Gamma(\Xi_b) - \Gamma(\Xi_b \rightarrow \Lambda_b + X). \quad (15)$$

Using the preliminary lattice values [37] for the matrix elements of Λ_b we obtain

$$\frac{\bar{\tau}(\Xi_b^0)}{\bar{\tau}(\Xi_b^+)} = 1 - 0.12 \pm 0.02 \pm ???, \quad (16)$$

where ??? stands for some unknown systematic errors. As a further approximation we equate $\bar{\tau}(\Xi_b^0)$ to $\tau(\Lambda_b)$ - here similar cancellations arise as in τ_{B_s}/τ_{B_d} -, so we arrive at the following prediction

$$\frac{\tau(\Lambda_b)}{\bar{\tau}(\Xi_b^+)} = 0.88 \pm 0.02 \pm ???. \quad (17)$$

Mixing

The mixing quantities have been investigated in detail in [4], numerically we obtain

$$\Delta M_d = 0.53 \pm 0.18 \text{ ps}^{-1}, \quad \Delta M_s = 19.3 \pm 6.7 \text{ ps}^{-1}, \quad (18)$$

$$\Delta \Gamma_d = (2.67_{-0.65}^{+0.58}) \cdot 10^{-3} \text{ ps}^{-1}, \quad \Delta \Gamma_s = 0.096 \pm 0.039 \text{ ps}^{-1}, \quad (19)$$

$$\Delta \Gamma_d / \Gamma_d = (4.09_{-0.99}^{+0.89}) \cdot 10^{-3}, \quad \Delta \Gamma_s / \Gamma_s = 0.147 \pm 0.060, \quad (20)$$

$$\Delta \Gamma_d / \Delta M_d = (52.6_{-12.8}^{+11.5}) \cdot 10^{-4}, \quad \Delta \Gamma_s / \Delta M_s = (49.7 \pm 9.4) \cdot 10^{-4}, \quad (21)$$

$$\phi_d = -0.091_{-0.038}^{+0.026}, \quad \phi_s = (4.2 \pm 1.4) \cdot 10^{-3}, \quad (22)$$

$$a_{fs}^d = (-4.8_{-1.2}^{+1.0}) \cdot 10^{-4}, \quad a_{fs}^s = (2.06 \pm 0.57) \cdot 10^{-5}. \quad (23)$$

The predictions for $\Delta \Gamma_d$ and $\Delta \Gamma_d / \Gamma_d$ are obtained [4] under the assumption that there are no new physics contributions in ΔM_d . From this list one sees the strong suppression of ϕ and a_{sl} in the standard model.

EXPERIMENTAL STATUS

Lifetimes

The Heavy Flavor Averaging Group quotes [40] the following numbers

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.071 \pm 0.009, \quad \frac{\tau(B_s)}{\tau(B_d)} = 0.939 \pm 0.021, \quad (24)$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.921 \pm 0.036, \quad \tau(B_c) = 0.463 \pm 0.071 \text{ ps}. \quad (25)$$

From the ratio τ_{B^+}/τ_{B_d} it can be seen that the HQE works very well. τ_{B_s}/τ_{B_d} is about 2.9σ below 1, here more precise numbers are needed, to see whether there might be some interesting effects. The situation for the Λ_b -baryon is not settled yet. First several theoretical improvements have to be included, second there are two different experimental numbers on the market [41, 42]. For B_c the number lies in the right

ball park, but here also a full NLO-QCD calculation would be desirable to make the comparison more quantitative. Finally we are waiting for a first result for the lifetimes of the Ξ_b -baryons.

Mixing

The mass differences have been measured with great precision at LEP, TeVatron and the B factories [43, 44, 45, 40]

$$\Delta M_d = 0.507 \pm 0.005 \text{ ps}^{-1}, \quad (26)$$

$$\Delta M_s = 17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1}. \quad (27)$$

Due to the uncertainties in the decay constants, theory will not be able to achieve a similar accuracy in the foreseeable future.

For the remaining mixing quantities a_{sl} , $\Delta\Gamma$ and ϕ we do not have measurements yet, but very interesting bounds:

In [46, 47] the dimuon-asymmetry was determined

$$a_{sl} = 0.582 a_{sl}^d + 0.418 a_{sl}^s, \quad (28)$$

$$a_{sl}^{\text{D0}} = (-5.3 \pm 2.5 \pm 1.8) \cdot 10^{-3}, \quad (29)$$

$$a_{sl}^{\text{CDF}} = (+8.0 \pm 9.0 \pm 6.8) \cdot 10^{-3}. \quad (30)$$

The semileptonic CP asymmetry was also measured directly in [48].

$$a_{sl}^s = (1.23 \pm 0.97 \pm 0.17) \cdot 10^{-2}. \quad (31)$$

Here more precise numbers are needed, because a clear deviation from the small standard model value would be an unambiguous sign for new physics! The same argument holds for the phase ϕ_s , while a clean measurement of $\Delta\Gamma$ is probably best exploited by comparing experiment and theory for $\Delta\Gamma/\Delta M$.

$\Delta\Gamma$ and ϕ_s have been determined from an angular analysis in the decay $B_s \rightarrow J/\psi\phi$: In the untagged analysis from D0 [49] the following values were obtained

$$\phi_s = 0.79 \pm 0.56_{-0.01}^{+0.14}, \quad (32)$$

$$\Delta\Gamma = 0.17 \pm 0.09 \pm 0.02 \text{ ps}^{-1}. \quad (33)$$

One has to keep in mind the 4-fold ambiguity in ϕ_s : with ϕ_s also $-\phi_s$ and $\pi \pm \phi_s$ are solutions! CDF obtained from the untagged analysis [50]

$$\Delta\Gamma = 0.076_{-0.063}^{+0.059} \pm 0.006 \text{ ps}^{-1}. \quad (34)$$

and no bound on ϕ_s .

CDF also performed a tagged analysis [51] and obtains confidence regions in the $\phi_s - \Delta\Gamma$ -plane, which differ about 1.5σ from the SM prediction. If they fix $|\Gamma_{12}|$ to the SM value obtained in [4] they get

$$-\phi_s \in [0.24, 1.36] \cup [1.78, 2.90]. \quad (35)$$

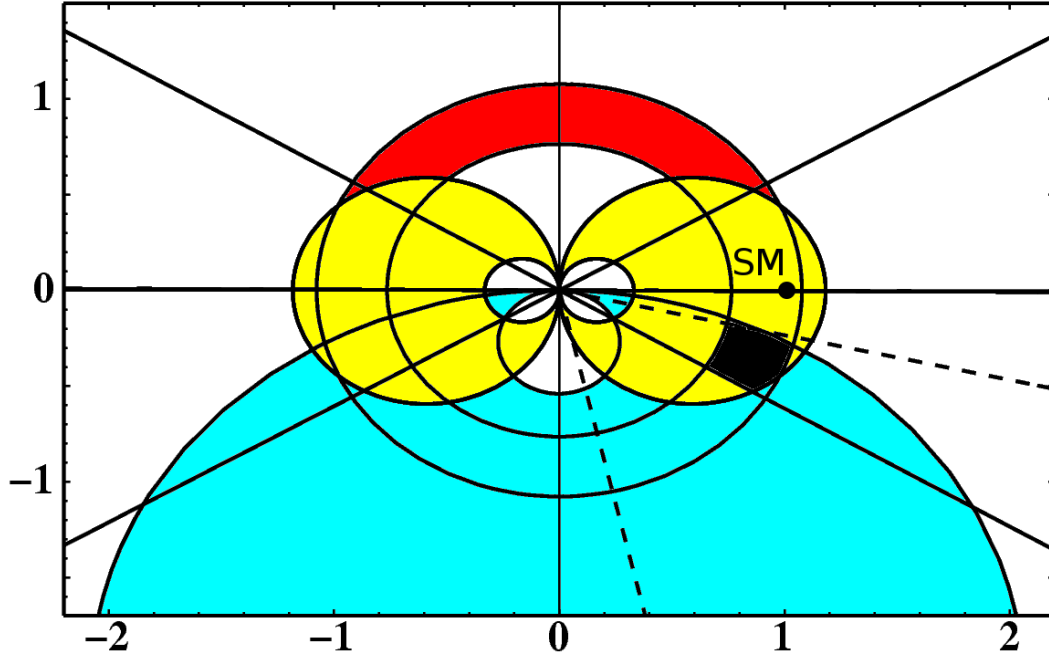


FIGURE 1. Experimental bounds in the complex Δ_s -plane (state: end of 2006). The bound from ΔM_s is given by the red (dark-grey) ring around the origin. The bound from $\Delta\Gamma_s/\Delta M_s$ is given by the yellow (light-grey) region and the bound from a_{fs}^s is given by the light-blue (grey) region. The angle ϕ_s^Δ can be extracted from $\Delta\Gamma_s$ (solid lines) with a four fold ambiguity - one bound coincides with the x-axis! - or from the angular analysis in $B_s \rightarrow J/\psi\phi$ (dashed line). If the standard model is valid all bounds should coincide in the point (1,0). The current experimental situation shows a small deviation, which might become significant, if the experimental uncertainties in $\Delta\Gamma_s$, a_{st}^s and ϕ_s will go down in near future.

NEW PHYSICS MODELS

In the literature many new physics models are applied to the mixing sectors, e.g. [52, 53] and references in [4]. In [4] we have presented a model independent way to determine new physics effects in the mixing sector. We assume that new physics does not alter Γ_{12} - at least not more than the intrinsic QCD uncertainties, but it might have a considerable effect on M_{12} . Therefore we write

$$\Gamma_{12} = \Gamma_{12}^{\text{SM}} \quad M_{12} = M_{12}^{\text{SM}} \cdot \Delta \quad (36)$$

By comparing experiment and theory for the different mixing observables we get bounds in the complex Δ -plane, see [4]. Taking the solution for ϕ_s from the untagged D0 analysis in the 4th quadrant - which corresponds to a certain choice of the strong phases in the decay $B_s \rightarrow J/\psi\phi$ - and the data that were available at the end of 2006, we obtained in [4] a 2σ deviation from the standard model, see Fig. (1). A new analysis is currently in progress.

Note added: (taken from [54]) There is sometimes a confusion between the mixing phases β_s and ϕ_s , which we would like to address here. Both numbers are expected to be small in the standard model - $\phi_s = (0.24 \pm 0.04)^\circ$ and $2\beta_s = (2.2 \pm 0.6)^\circ (=$

$(0.04 \pm 0.01)\text{rad}$), but in view of the high future experimental precisions - in particular at LHCb [2]- a clear distinction might be useful.

$2\beta_s := -\arg[(V_{tb}V_{ts}^*)^2/(V_{cb}V_{cs}^*)^2]$ is the phase which appears in $b \rightarrow c\bar{c}s$ decays of neutral B-mesons taking possible mixing into account, so e.g. in the case $B_s \rightarrow J/\psi + \phi$. $(V_{tb}V_{ts}^*)^2$ comes from the mixing (due to M_{12}) and $(V_{cb}V_{cs}^*)^2$ comes from the ratio of $b \rightarrow c\bar{c}s$ decay and $\bar{b} \rightarrow \bar{c}c\bar{s}$ amplitudes. Sometimes β_s is approximated as $2\beta_s \approx -\arg[(V_{tb}V_{ts}^*)^2] \approx -\arg[(V_{ts}^*)^2]$ - the error due to this approximation is on the per mille level.

$\phi_s := \arg[M_{12}/\Gamma_{12}]$ is the phase that appears e.g. in a_{fs}^s . In M_{12} we have again $(V_{tb}V_{ts}^*)^2$, while we have a linear combination of $(V_{cb}V_{cs}^*)^2$, $V_{cb}V_{cs}^*V_{ub}V_{us}^*$ and $(V_{ub}V_{us}^*)^2$ in Γ_{12} . Neglecting the latter two contributions - which is not justified - would yield the phase $2\beta_s$.

New physics alters the phase $-2\beta_s$ to $\phi_s^\Delta - 2\beta_s$ and the phase ϕ_s to $\phi_s^\Delta + \phi_s$. If the new physics contribution is sizeable, then in both cases only ϕ_s^Δ survives, since the standard model phases are very small.

In the tagged analysis CDF [51] introduces the phase $2\beta_s$ for which the following relation to the notion in [4] holds $-2\beta_s := \phi_s^\Delta - 2\beta_s^{SM}$.

OUTLOOK

In this talk we have summarized the current theoretical status of the lifetimes of b -hadrons and the mixing quantities. Our main strategy for finding new physics in these quantities is the following: New physics is expected to have the biggest effects in M_{12} , but due to the decay constant problem the quantity that comes first in mind - ΔM - seems to be not the best choice. We have argued that Γ_{12}/M_{12} is theoretically very well under control. Therefore our first choice are the quantities $\Delta\Gamma/\Delta M$, a_{sl} and ϕ . Moreover the theoretical precision in the determination of Γ_{12} can be tested directly by investigating the lifetimes of b -hadrons, because both quantities rely on the same theoretical footing.

We conclude with a subjective wish-list for theory and experiment:

- Perturbative calculations:
 - NLO-QCD corrections for $\tau(B_c)$
 - complete NLO-QCD corrections for $\tau(\Lambda_b)$
 - $\Gamma_4^{(1)}$ for Γ_{12}
 - $\Gamma_3^{(2)}$ for Γ_{12}
- Non perturbative calculations:
 - matrix elements for $\tau(B^+)/\tau(B_d)$
 - matrix elements for $\tau(\Lambda_b)$ and $\tau(\Xi_b)$
 - precise and reliable values for the decay constants
 - $1/m$ -operators for Γ_{12} , a first step in that direction has been performed in [55]
- Experiment: (ranked)
 - 1) Precise values for a_{sl} and ϕ_s
 - 2) Precise values for $\tau(B_s)/\tau(B_d)$ and $\tau(\Lambda_b)$ and $\Delta\Gamma$ — a first value for $\tau(\Xi_b)$

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APPENDIX

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